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Note on Anomaly Cancellation on $SO(32)$ heterotic 5-brane

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Abstract

We show that the gauge, gravitational (tangent-bundle) and their mixed anomalies arising from the localized modes near a 5-brane in the $SO(32)$ heterotic string theory cancel with the anomaly inflow from the bulk with the use of the Green-Schwarz mechanism on the brane, similarly to the $E_8 \times E_8$ 5-brane case. We also compare our result with Mourad's analysis performed in the small-instanton limit.

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One of the most amazing aspects of string theory is the miraculous mechanism of anomaly cancellation. In the ten-dimensional bulk, the anomalies of $N = 1$ superstrings are successfully cancelled by the use of the well-known Green-Schwarz mechanism [1], in which the two-form B field is assumed to change with respect to the gauge and local Lorentz transformations. Anomalies of chiral matter fields supported on some branes are also known to cancel with inflow contributions from the bulk [2]. In this letter, we focus on the gauge, gravitational (tangent-bundle) and their mixed anomalies arising from the localized modes near a 5-brane in the $SO(32)$ heterotic string theory. We show that their anomalies also cancel with the anomaly inflow from the bulk with the use of the Green-Schwarz mechanism on the brane, similarly [3] to the case of the $E_8 \times E_8$ 5-brane. Although the argument that the anomalies on a heterotic 5-brane should cancel with an anomaly inflow is an old one [4], the arithmetic we show below is new and different from [4], as, for instance, we do not consider any "current at infinity". We also compare our result with Mourad's analysis [5] in which the small-instanton limit was considered. Anomaly cancellation on heterotic 5-branes in the K3 compactification was discussed in [6].

Let us start with the symmetric 5-brane solution [7, 8] in the $SO(32)$ heterotic string theory. It has been known for some time that the moduli of this solution consists of $D = 6$, $\mathcal{N} = 1$ 30 hypermultiplets [9]. The bosonic moduli are four Nambu-Goldstone modes associated with the spontaneously broken translational invariance, one scale modulus and 115 moduli coming from the arbitrariness of the choice of $SU(2)$ subgroup of $SO(32)$ in which the spin connection is embedded. The number of 115 can be easily counted by the decomposition of $SO(32)$ in terms of $SO(28) \times SU(2) \times SU(2)$ as follows:

$$\mathbf{496} \quad = \quad (\mathbf{378}, \mathbf{1}, \mathbf{1}) \otimes (\mathbf{1}, \mathbf{3}, \mathbf{1}) \otimes (\mathbf{1}, \mathbf{1}, \mathbf{3}) \otimes (\mathbf{28}, \mathbf{2}, \mathbf{2}). \tag{1}$$

Suppose that the $SU(2)$ spin connection is embedded into the last $SU(2)$ subgroup. Then the centralizer $SO(28) \times SU(2)$ remains as the unbroken gauge group, while the rest of $3 + 28 \times 2 \times 2 = 115$ generators give rise to deformations, being moduli of this solution.

Thus, 28 of the 30 hypermultiplets, which contain 56 symplectic Majorana-Weyl spinors, transform as $(\mathbf{28}, \mathbf{2})$ with respect to the unbroken $SO(28) \times SU(2)$ gauge symmetry, while the remaining two are gauge singlets. Anomaly polynomials for the chiral fermions belonging

to these hypermultiples are:

$$\begin{aligned}
I_8^{(\mathbf{28},\mathbf{2})} &= \frac{1}{2}\hat{A}(T\Sigma)\cdot \mathrm{tr}_{(\mathbf{28},\mathbf{2})}e^{iF}\bigg|_8 \\
&= \frac{28}{5760}(-4p_2+7p_1^2)+\frac{1}{96}p_1\mathrm{tr}_{(\mathbf{28},\mathbf{2})}F^2+\frac{1}{48}\mathrm{tr}_{(\mathbf{28},\mathbf{2})}F^4, \qquad (2)\\
I_8^{singlet} &= \frac{1}{2}\hat{A}(T\Sigma)\bigg|_8 \times 4 \\
&= \frac{2}{5760}(-4p_2+7p_1^2), \qquad (3)
\end{aligned}$$

where $\hat{A}(T\Sigma)$ is the Dirac genus of the tangent bundle of the 5-brane and F is the 2-form for the $SO(28)\times SU(2)$ gauge field strength. Similarly to [3], we have ignored the normal bundle anomalies in (2) and (3). The total anomaly I_6^1 is obtained by the well-known descent relations:

$$\begin{aligned}
I_8 &= I_8^{(\mathbf{28},\mathbf{2})}+I_8^{singlet}, \qquad (4)\\
I_8 &= dI_7, \qquad (5)\\
\delta I_7 &= dI_6. \qquad (6)
\end{aligned}$$

On the other hand, the bulk supergravity action contains the Green-Schwarz counterterm proportional to BX_8 with

$$X_8 = \frac{1}{24}\left(\frac{1}{8}\mathrm{tr}R^4+\frac{1}{32}(\mathrm{tr}R^2)^2-\frac{1}{240}\mathrm{tr}R^2\mathrm{Tr}F_{SO(32)}^2+\frac{1}{24}\mathrm{Tr}F_{SO(32)}^4-\frac{1}{7200}(\mathrm{Tr}F_{SO(32)}^2)^2\right), \qquad (7)$$

where $F_{SO(32)}$ is the 2-form for the $SO(32)$ gauge field strength and Tr is the trace in the adjoint **496** representation. R is the curvature 2-form for the ten-dimensional bulk tangent bundle Q, which is decomposed, in the presence of the 5-brane, into a direct sum of the tangent bundle of the brane, $T\Sigma$, and the normal bundle N of it. A Pontryagin class of Q can be expressed as a polynomial of Pontryagin classes of $T\Sigma$ and N. However, since we have taken only the tangent bundle anomalies into account in (2) and (3), we may identify the curvature 2-form R for the total bundle Q to be the curvature 2-form for the tangent bundle $T\Sigma$ of the brane, and examine the cancellation of the tangent bundle anomalies, as well as the gauge anomalies, and the mixed ones for the gauge and tangent bundles. In the end of this note, we also comment on the normal bundle anomaly cancellation in the present setting. With these remarks we write X_8 in terms of traces of the subgroup $SO(28)\times SU(2)$:

$$X_8 = \frac{1}{192}\left(3p_1^2-4p_2+2p_1(\mathrm{tr}_{\mathbf{28}}F_{SO(28)}^2+2\mathrm{tr}_{\mathbf{2}}F_{SU(2)}^2)+8(\mathrm{tr}_{\mathbf{28}}F_{SO(28)}^4+2\mathrm{tr}_{\mathbf{2}}F_{SU(2)}^4)\right), \qquad (8)$$

where p_1 and p_2 are now understood as the Pontryagin classes for $T\Sigma$.

The gauge and local Lorentz variations of the B field in the $BX_8 \sim -dBX_7$ term precisely cancel the ten-dimensional bulk anomalies in the $SO(32)$ string theory; this is the Green-Schwarz mechanism. On the other hand, if there is no brane, the variations of X_7 vanishes because $d^2B = 0$. However, since the 5-brane is a magnetic source for the B field, the variations of X_7 give rise to, in the presence of the 5-brane, δ-function-like contributions on the 5-brane known as anomaly inflows. Therefore, the total anomalies are described by the invariant polynomial $I_8^{(\mathbf{28},\mathbf{2})} + I_8^{singlet} - X_8$, which turns out, using (2), (3) and (8), to factorize as

$$I_8^{(\mathbf{28},\mathbf{2})} + I_8^{singlet} - X_8 = -\frac{1}{96}\left(\mathrm{tr}R^2 - \mathrm{tr}_{\mathbf{32}}F^2_{SO(32)}\right)\left(p_1 + 12\mathrm{tr}_{\mathbf{2}}F^2_{SU(2)}\right). \tag{9}$$

Here we have reexpressed $\mathrm{tr}_{\mathbf{28}}F^2_{SO(28)} + 2\mathrm{tr}_{\mathbf{2}}F^2_{SU(2)}$ as the $SO(32)$ fundamental trace in the first parentheses. The first factor is precisely the combination that appears in the anomalous Bianchi identity of the H field, and therefore the sum of anomalies can be cancelled by introducing a local counterterm on the 5-brane, similarly to the cases of the type I [5] and $E_8 \times E_8$ [3] 5-branes.

Let us compare the arithmetic we presented above with the known mechanism of anomaly cancellation on the $SO(32)$ 5-brane in the small-instanton limit [5]. If the instanton size of the heterotic 5-brane tends to zero, the theory becomes strongly coupled and the supergravity analysis loses its validity. It was proposed that there would then be an enhanced $Sp(k)$ gauge symmetry [10] on k parallel 5-branes in this limit of the $SO(32)$ theory, in addition to the full $SO(32)$ gauge symmetry. This argument was supported by the proof of anomaly cancellation in the S-dual type I brane system [5]: The dual type I D5-branes have three kinds of zeromode hypermultiplets, called θ, λ and ψ in [5]; they transform as $(\mathbf{4}_+, \mathbf{2}_+, \mathbf{1}, \mathbf{1})$, $(\mathbf{4}_-, \mathbf{2}_-, \mathbf{1}, \mathbf{3})$ and $(\mathbf{4}_+, \mathbf{1}, \mathbf{32}, \mathbf{2})$, respectively, under the actions of $SO(5,1) \times SO(4) \times SO(32) \times SU(2)$, where the first two factors are the ten-dimensional Lorentz group, while the last $SU(2) = Sp(1)$ is the enhanced gauge symmetry. The subscripts $\pm$ denote the chiralities of the spinors.

Let I_8^θ, I_8^λ and I_8^ψ be the anomaly polynomials of θ, λ and ψ, respectively. If $k = 1$, the total anomaly turns out to be [5]

$$I_8^\theta + I_8^\lambda + I_8^\psi - X_8 = -\frac{1}{96}\left(\mathrm{tr}R^2 - \mathrm{tr}_{\mathbf{32}}F^2_{SO(32)} + 4\chi(N)\right)\left(p_1(Q) + 12\mathrm{tr}_{\mathbf{2}}F^2_{SU(2)} - 2p_1(N)\right), \tag{10}$$

where $\chi(N)$ is Euler class of the normal bundle.

If all the terms depending on the normal-bundle connections are ignored in (10), then $p_1(Q)$ is replaced with $p_1(T\Sigma)$, and (10) looks superficially the same as (9). There are,

however, a number of significant differences between our result and Mourad's analysis as follows:

(i) In (9), $F_{SU(2)}$ is the field strength of the unbroken $SU(2)$ *subgroup* of $SO(32)$, whereas in (10) is that of the *enhanced* $SU(2)$ gauge group which is independent of the bulk $SO(32)$ gauge symmetry.

(ii) Although one could decompose $SO(32)$ representations into those of the subgroup $SO(28)\times SU(2)$ in (10), the supermultiplets turn out to transform quite differently from those in our "broken" case. For instance, we have an $SO(28) \times SU(2)$ bifundamental, while there arise no such representations in the small-instanton case.

(iii) Mourad's proof of cancellation extends to $k(\geq 2)$ $D5$-branes, while it is not obvious to generalize our argument to the case of many heterotic 5-branes.

(iv) It is also difficult to include the normal-bundle contributions in our case; a naive inclusion of them does not lead to the desired factorized form. Since it is known that the mechanisms of normal bundle anomaly cancellation on M5-branes requires a complicated setting [11], we might also need to consider in heterotic string theories such a modification of the solution without small instantons.

Since the heterotic/type I duality is a strong-weak duality, there is no guarantee that how anomalies cancel in one theory will be the same in the other theory. Therefore, we conclude that the superficial similarity between (9) and (10) will be an accident for $k = 1$.

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